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ABSTRACT

This paper investigates one high school student's use of gestures in an interview context in which he worked on the problem of understanding graphical representations of motion. The goal of the investigation was to contribute a detailed analysis of the process of learning as it occurred over a short time period in order to contribute to the broader understanding of the role of the body in the development of new knowledge in mathematics. In particular, the paper addresses the following questions: (1) How might students make the link between powerful understanding of visual attributes of graphs and the bodily experiences of creating graphs of their own motions?; and (2) How can the dynamic nature of gestures help students understand the dynamic nature of graphs of motion? (SOE)

Gesture and the Mathematics of Motion

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Introduction

While many of us first learned mathematics as a practice of symbol manipulation with paper and pencil, educational technologies developed in recent decades, such as computer software, computer-driven interactive devices (Clements, Nemirovsky, & Sarama, 1995; Confrey, 1991; Kaput, 1997; Nemirovsky, ; The_Visual_Geometry_Project, 2002), as well as a wide range of manipulative materials, have increased the diversity ways in which students can learn and do mathematics. Research on student learning in these new environments, and in more traditional learning environments, has highlighted the importance of spatial reasoning (Jacobson & Lehrer, 2000; Outhred & Mitchelmore, 2000), visualization (Nemirovsky & Noble, 1997; Noble, Nemirovsky, DiMattia, & Wright, 2002; Presmeg, 1992; Zaskis, Dubinsky, & Dautermann, 1996), physical motion (Nemirovsky, Tierney, & Wright, 1998; Noble & Nemirovsky, 1995), and kinesthetic perception (Nemirovsky, 1993; Nemirovsky et al., 1998; Noble & Nemirovsky, 1995; Noble, Nemirovsky, Wagoner, Solomon, & Cook, 1996) for learning mathematics.

These studies, and the educational technology that has spurred them on, have led to a growing awareness of mathematics learning as an activity that includes both the body and the mind. Studies in both mathematics and science education have shown the importance of how one places oneself physically in relationship to mathematical objects or representations (Nemirovsky & Monk, 2000; Roth, submitted); of the experience of moving in order to produce a given graphical shape (Nemirovsky et al., 1998; Noble & Nemirovsky, 1995; Noble, Nemirovsky, Wright, & Tierney, 2001; Thornton & Sokoloff, 1989); and of the experience of enacting physical phenomena such as the path of rays of sunlight (Crowder, 1996), or the pull of forces on a pulley system (Roth, submitted). Each of these is an example of learning in which the thinking through of a problem is in part a bodily activity.

Traditional cognitivist models of the brain as a computing device, strictly separated from the body, worked well for describing the kinds of symbol-manipulation tasks that at one time dominated mathematics classrooms. However, new models are needed to understand the wide range of ways in which today's students engage with mathematics. The work of researchers who study "embodied cognition" (Freeman & Nuñez, 1999; Iverson & Thelen, 1999; Nuñez, 1999; Rosch, 1999) calls for understanding thinking as a bodily achievement and concepts as inevitably and inextricably tied to humans' experiences of being bodily in the world.

At the same time as awareness of the importance of the body in cognition has been growing, the study of gestures, or "the wide variety of ways in which humans give what is usually regarded as willful expression to their thoughts and feelings through visible bodily action," (Kendon & Müller, 2001, p. 1) has also been developing. Cross-disciplinary interest in the gestures of humans has recently reached the threshold required to launch a new journal devoted exclusively to research on gestures (Gesture, 2001), and to initiate an international conference on gesture (Gesture-The Living Medium: First Congress of the International Society for Gesture Studies, University of Texas at Austin, June 2002).

The development of the field of gesture studies provides an opportunity for educators to use what has been learned about human gestures to help us to better understand the role of the body in thinking and learning. Studies of children's gestures have included descriptions of their gestures while re-telling stories (Riseborough, 1982), describing how to play a board game (Evans & Rubin, 1979), giving spatial directions (Iverson & Goldin-Meadow, 1997), doing conservation tasks (Church & Goldin-Meadow, 1986), and solving arithmetic and word problems (Alibali, Bassok, Solomon, Syc, & Goldin-Meadow, 1999; Alibali & Goldin-Meadow, 1993). Among the provocative findings of these studies is the consistent observation that some children convey different information in speech than in gesture. Even more surprisingly, when there was a mismatch (Alibali & Goldin-Meadow, 1993) between speech and gesture, the gesture generally conveyed the more correct information (Church & Goldin-Meadow, 1986; Evans & Rubin, 1979). In addition, Alibali and Goldin-Meadow (1993) found that children tended to pass through a transitional stage of mismatched gesture and speech as they learned how to solve addition and multiplication problems. They found that children who did not pass through this stage tended to perform less well on posttests than children who did pass through a mismatching stage. All of these results point out the importance of attending to students' gestures as we attempt to understand their learning processes.

These findings have led several researchers to posit a role for gestures in the processes of thinking and learning (Alibali, Kita, & Young, 2000; Goldin-Meadow, 2001). Some have proposed that gestures help the gesturer to 'package' information, especially spatial information, so that it can be described in words (Alibali et al., 2000). For example, children in one study used gestures in a conservation task to indicate the relevant dimensions of two pieces of playdough before verbally explaining the relationship of the two pieces. Science educators have found evidence suggestive of additional uses of gesture in thinking and learning when students are working on science tasks. Students have been found to use gestures to convey or try out ideas that they cannot yet describe using speech (Crowder, 1996; Roth, 2000) to create and run models of physical phenomena such as the travel of rays of sunlight (Crowder, 1996), and to make invisible phenomena visible (Noble, Rizzuto, & Goldstein, 2002).

The findings of science educators suggest that additional uses of gestures may be found by looking at students' work on mathematics tasks which call for different kinds of models and uses of imagination. In this paper, I will investigate one high school student's use of gestures in an interview context in which he worked on the problem of understanding graphical representations of motion. The goal of this investigation is to contribute a detailed analysis of the processes of learning as they occur over a short time-scale, in order to contribute to the broader project better understanding the role of the body in the development of new knowledge in mathematics. In particular, this paper addresses the questions: How might students make the link between powerful understandings of visual attributes of graphs and their bodily experiences of creating graphs of their own motions? and How can the dynamic nature of gestures help students to understand the dynamic nature of graphs of motion?

Graphs and Body Motion

Many researchers, curriculum developers, and teachers have also experimented with graphing physical phenomena as a way to link students' bodily experiences with graphical shapes. Since the dawn of MBL technology (Mokros & Tinker, 1987), the use of the motion detector to create graphs has become extremely popular in technology-rich science and mathematics classrooms. Researchers have described the way that students' physical activity and past experiences moving in the world can become resources for understanding graphical shapes, when they use tools which link physical motion with graphical shape (Meira, 1995; Moschkovitch, 1996; Nemirovsky & Noble, 1997; Nemirovsky et al., 1998; Noble, Nemirovsky, Tierney, & Wright, 1999; Noble et al., 2001). Graphs created by a motion detector in response to a student's own motion embody a link between a mathematical representation and body motion. Thus, the use of the motion detector provides a useful context for exploring this connection, and may suggest directions for future research on how this connection is made in other kinds of mathematical tasks.

While much of the research on students' use of motion detectors has involved the creation of temporal graphs of distance vs. time or velocity vs. time (Nemirovsky et al., 1998; Thornton & Sokoloff, 1990), this article concerns the work of one student to understand the relation of velocity vs. position graphs to his own motions. These graphs are an important part of dynamical systems modeling (Tufillaro, Abbott, & Reilly, 1992), in part because they represent the state of a moving object (given by its position and velocity) as a single point on the graph, creating a compact representation of a system's behavior, in which the relationship between the velocity and position of an object determines the graph shapes. For example, the simple harmonic motion of a weight bouncing up and down on a hanging spring, assuming no damping for simplicity, could be represented as a sine wave on a position vs. time graph that evolves to the right for as long as the motion lasts, or as an ellipse on a v vs. p graph, drawn over and over as the weight continues to bounce (See Figure 1).

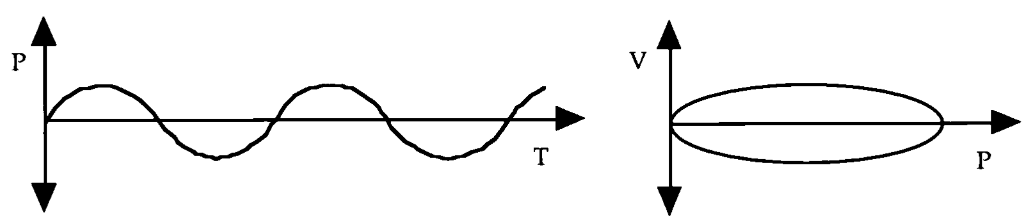


Figure 1

As we see in Figure 1, the constraints on graph shapes in velocity vs. position space are different than those we are accustomed to in temporal graphs. A velocity vs. position graph can double back, or "go backwards," because position can decrease as well as increase. Thus, unlike temporal graphs, graphs of velocity vs. position are not functions, and have a different logic than temporal graphs do.

These velocity vs. position graphs provide an exceptional opportunity to investigate the role of one's own motion in graph interpretation, because the dynamics of these graphs, the way they unfold in time, is not explicitly represented in the form of a time axis. One must infer the dynamics of the graph, or how it is drawn over time, from the values present in the graph. Other researchers who have studied the use of non-temporal graphs (Ochs, Gonzales, & Jacoby, 1996; Ochs, Jacoby, & Gonzales, 1994) have found that the scientists they studied used their own motions to trace out pathways in these graphs in order to add the dimension of time back into these graphs. This investigation of a student using phase space graphs will yield examples of similar uses of body motion to imagine the dynamics of these graphs.

Method

The data analyzed in this article comes from a series of five hour-long individual teaching experiments (Cobb & Steffe, 1983) with one student. The interviewer posed some pre-determined problems to the student, but also encouraged the students to explore questions of their own whenever possible. The videotape of each interview was analyzed in order to plan for the next interview. After the interviews were completed, all interviews were divided into episodes consisting of coherent units of work within the larger interview, and each episode was summarized. Selected episodes were transcribed, and two researchers reviewed all of these episodes, and selected a small number of episodes for further detailed analyses. The episodes in this article are among those so selected.

The Episodes

The episodes which follow come from the first interview the author did with a student who we call Noam[1]. At the time of this interview, Noam was a high school senior at a suburban vocational and technical high school connected to a traditional academic high school. Noam had been primarily in the vocational track, and he did vocational work and academic work on alternating weeks. He had taken two years of math: arithmetic and a pre-algebra/algebra class, and one year of general science at the time of the interview. At the time of the interview, he was enrolled in a physics class which involved a limited amount of algebra-level mathematics.

The interviewer and Noam spent the first half of the first interview making drawings to represent their motions of a hand-held toy car, and trying to act out each other's drawings by moving the car. Halfway into the interview, the interviewer introduced the motion detector to Noam, with the minimal explanation necessary for him to start using it. The motion detector senses the distance from itself to the nearest object in its path, and the software

[1] This is a pseudonym.

(MacMotion™) uses this information to compute the velocity of the object in real time. The motion detector was set up at one end of a long table (hereafter called the car table), facing a tape line along one table, which was intended to help to guide the student's motion along a straight line. A small plastic car was placed in front of the motion detector, with a paper card affixed to the front of it, to provide a single location that could be sensed by the motion detector. A second table (hereafter called the computer table), placed at the end of the first, held the computer to which the motion detector was sending data (See Figure 2).

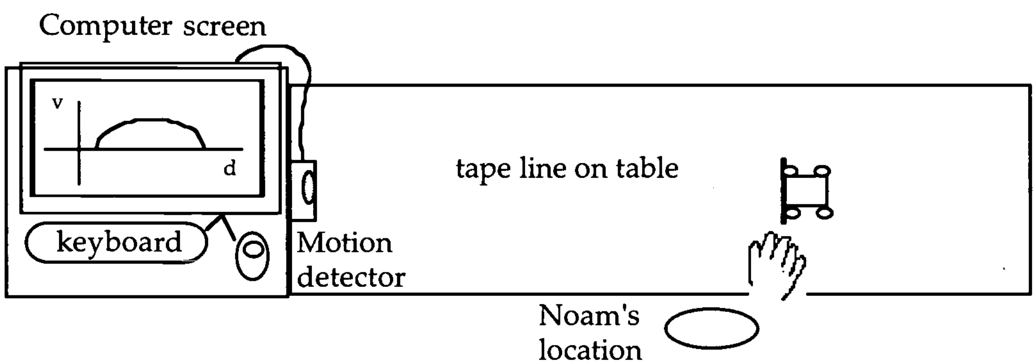


Figure 2

Before beginning work with the motion detector, Noam was told that the small plastic box would detect his motion, and that he should move in front of it to make a graph on the computer screen. Before Episode 1, Noam made his first motion in front of the motion detector, but ran out of time before he completed it. Noam decided to try the motion again, but faster.

<u>Transcript</u>

The following Episodes begin about halfway into Noam's first interview, and they occur in immediate succession.

<u>Transcript conventions</u>

In the table below, utterances are shown in the table column on the left and simultaneous gestures are shown in the table column on the right. Unless otherwise indicated, the gestures and words are co-extensive in time. If a gesture occurs during only part of the speech transcribed, then that part of the speech is underlined.

...   -indicates the voice trailing off.
. . .   -indicates omitted transcript.
:   -indicates an elongated vowel within a word, such as "wo:rd"
italics   -in transcript indicate emphasized words.
points to   -indicates a pointing motion of the hand rather than the cursor.
[*square brackets*] enclose referents for pronouns, pauses [*pause*], and notation for inaudible speech [*inaud.*].

<u>Episode 1 -- "How come it's [Graph 1] not going in a straight line. . . like I did?"</u>

00:35:50 - 00:38:00

Overview. In this Episode, Noam moves the toy car in front of the motion detector, causing a velocity vs. position graph for the motion to appear on the computer screen. The Interviewer then asks him to describe "what was happening where" on the graph, and Noam describes the graph as if it were the trajectory of the car, measured in two dimensions.

Text.
TRIAL: Noam moves the car at a slow, relatively constant speed, along the path represented below:

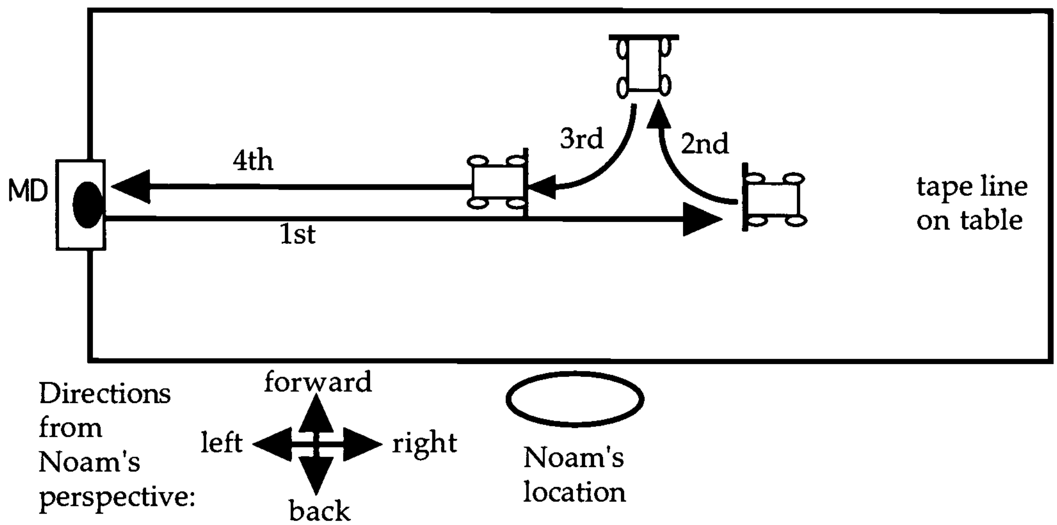
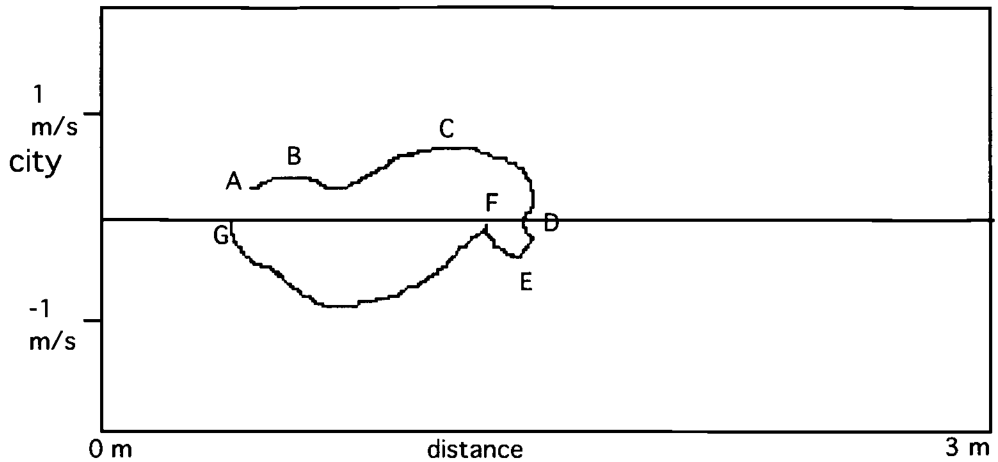


Figure 3

RESULT: The following graph appears on the computer screen:



Graph 1

Technical Notes:

• The letters indicating points A-F were added to the drawing for convenience of explanation.

• The graph can be interpreted by following the top half of the graph to the right, with the height of the graph indicating the speed at that position, so this indicates that the car was sped up a little (A to B), slowed down (between B and C), sped up more (to get to C), and slowed to a stop (C to D). This portion corresponds to Noam's 1st motion of the car to the right (See Figure 3) from about .5m to about 1.5m from the motion detector. The bottom half of the graph is in the negative velocity region, because any point on this graph has a negative value of velocity for its y-coordinate. This indicates that the car was moved back towards the motion detector again, that is, its distance from the motion detector was decreasing. Following this half of the graph from right to left (from larger to smaller distance from the motion detector), the graph suggests that the car was sped up a little (to the point E), then slowed to a stop (at F), which corresponds to Noam's 2nd motion of the car (See Figure 3). Continuing to read to the left, the graph indicates that the car was sped up to a large negative velocity , then slowed down and stopped at the starting point (G), which corresponds to Noam's 3rd and 4th motions (See Figure 3). This description breaks down in the immediate neighborhood of the point D[2].

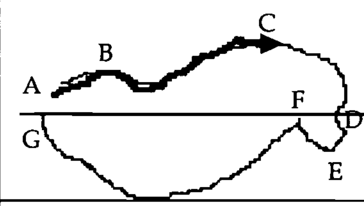
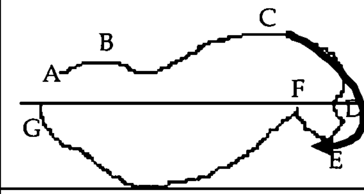
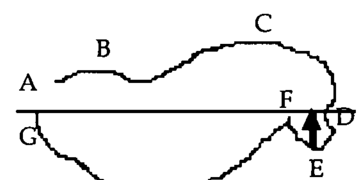
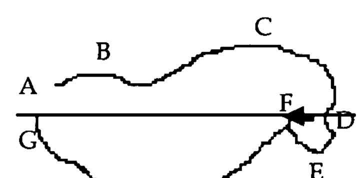
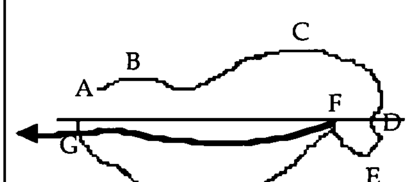
• From the above note and earlier descriptions, one can see that the motion detector reads only the distance an object is away from it, not its position in 2-dimensional space. The motion detector cannot know that Noam turned the car to the side in his 2nd motion (See Figure 3), only that he moved the car a short distance back towards the motion detector and then paused. We will see in the text that follows that Noam strove to make a connection between the 2-

[2]There is a problem with the graph at point D, where the line comes a little to the left above the axis, in the positive veloxity region, and moves a little to the right below the axis, in the negative velocity region. This is not a possible trajectory for a line produced by an actual motion. This reflects a limited malfunction of the motion detector graphing software, but since it is a small region, it does not make the graph as a whole uninterpretable.

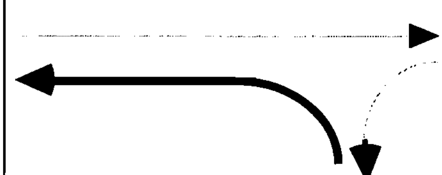
dimensional quality of his carefully performed motion and the 2-dimensional graph that was created as a result of this motion.

Tracy then invites Noam to sit in a chair in front of the computer table (See Figure 2).

<1> Int:	If you can just like walk me through that graph [Graph 1], telling me, um, what, you know, what was happening where on there,	*Points to the computer screen.*
<1.1> Int:	and you can just use this to point to different parts.	*Holds and moves the mouse.*
<2> Noam:	Okay. [*pause*] Where did I start?	*Takes mouse and moves the cursor on Graph 1 back and forth between A and D.*
<2.1>	I started from *this*, was it startin' from here?	*Points to A on Graph 1, looks toward the car table (the table on which he had moved the car).*
<3> Int:	I think you started from here, I think, I think, yeah, that's where you started.	*Points to A.*
<4> Noam:	OK. How come it's [*Graph 1*] not going in a straight line,	*`Runs finger across Graph 1 in a straight, horizontal line.*
<4.1>	like I did?	*Points to the car table. Turns toward Interviewer.*
<5> Int:	In a straight line like you did...	
<6> Noam:	Okay. Either way. Okay.	*Turns back toward the computer screen.*

<6.1>	I started coming here,	*Traces line from A to C with cursor, and turns toward Interviewer.* 
<7> Int:	Uh huh.	
<8> Noam:	It seems like I tu:rned,	*Traces this line with cursor:* 
<8.1>	[*pause*],	*Moves cursor from E to x-axis:* 
<8.2>	backed up,	*Moves cursor to F:* 
<8.3>	and started going over here.	*Traces this line with cursor:* 

<9> Int: Okay. And you were expecting it to be a straight line on top there?	*Points to Graph 1.*
<10> Noam: Over here, yeah.	*Runs finger across top half of Graph 1 on screen.*
<10.1> 'Cause I - I was going	*Moves hand in front of his body, as if holding the car and moving it in a straight line:* 
<10.2> straight on the line.	*Points to the line of tape on the table (see Figure 3), then resumes hand motion in space as if continuing straight motion of car for a short distance.*

<10.3>	And I turned,	*Moves hand in front of his body, as if moving the car.*  *Hand motion follows thick dark line; thin line indicates previous motion.* 
<10.4>	and started going back on the line again.	*Looks towards computer screen briefly. Moves hand along thick dark line, as if moving the car. Light lines show previous motions.* 
<10.5>	But they were off.	*Runs his finger horizontally across Graph 1 above the x- axis and then below the x-axis.*

Interpretive Notes. Noam had never used a graph of velocity vs. position before, and had limited experience with graphs of any kind. The Interviewer asked him to interpret a graph of velocity vs. position that had been created in response to his own motion, and Noam made a good initial guess at the meaning of this graph by linking the 2-dimensional motion of the car on the table with the 2-dimensional graph on the computer screen. For instance, Noam at first asked why the graph is not "going in a straight line like I did?" <4>. Two large segments of the motion of the car (See Figure 3) were along straight lines, and Noam was apparently expecting to see these straight paths reflected in the shape of the graph on the computer screen. Despite this discrepancy and the Interviewer's lack of explanation for it, Noam persevered in his attempt to describe how his motion correlated with various parts of the graph: "I started coming here [*moves cursor along graph from A to* C]," (<6.1>), relating his initial motion of the car to the graph line from A to C. Next, Noam said, "It seems like I tu:rned," <8>, elongating the vowel of the word "turned" while moving the cursor in a curved path from C to E. Noam's language suggests that the shape of the graph itself in this region, and the motion he made with the cursor along it, led him to believe that the segment from C to E was the location of a turn in his motion. Next Noam paused (<8.1>) while moving the cursor from point E up to the x-axis, continuing: "backed up [*moves the cursor to point F*]" (<8.2>). Noam's action during his pause preceded his assessment of this as a region in which he "backed up," suggesting that his action may have influenced his assessment of this region.

Despite the fact that Noam moved the cursor in the area of the graph line, his motions in <8.1> and <8.2> were not tracings of the graph line itself. Instead, it seems that here Noam used the graph as a frame in which to enact a "backing up" motion which resembled a part of his original trip with the car. His third motion (See Figure 4a) involved backing the car towards the tape line (Note that the card originally at the front of the car is facing backwards at the end of this motion.) and bringing it parallel to the tape line, as shown in the clip taken from Figure 3 and reproduced as Figure 4a below:

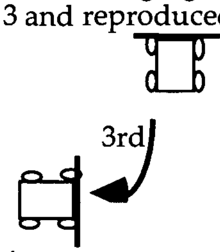


Figure 4a
detail from Figure 3

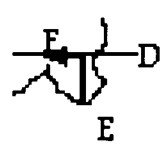


Figure 4b
detail of <8.1-2>

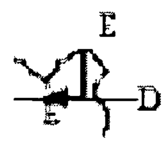


Figure 4c
detail flipped

Noam's motion of the cursor was apparently influenced by his memory of the motion he made with the car, as the motion he makes with the cursor: toward the axis and then across to F (See Figure 4b, above) is which is similar to the backing up motion he made with the car initially, though flipped over the horizontal axis (See Figure 4c, above).

Noam ended this description by saying, "and started going over here" (<8.3>) as he moved the cursor from the point F to the left beyond the point G, this time making no attempt to trace the graph itself, but simply taking an approximately straight path back to the starting point. This reflected Noam's actual motion of the car, which he moved straight back to the start after backing it onto the tape line again (See Figure 3).

In Noam's description of "what happened where" on the graph, he began by literally tracing the graph; moved toward inferring the motion of the car from the graph shape, as he said "It seems like I turned"; and finally described motions that owed more to the actual trip with the car than to the shape of the graph. He produced a reasonably accurate correlation of parts of the graph with his motions: the top half of the graph is rightward motion and the bottom half is leftward motion, and in between there is a "turn" or a change of direction. He had not yet found a way to explain the discrepancy between the shape of his motion and the shape of the lines of the graph.

It is important to note that not only was Noam's interpretation of the graph influenced by his remembered motion, but his memory of his motion was shaped by his use of the graph. The motion Noam ends up describing for the car in <10.1> - <10.4> is nearly identical to the motion he actually made with the car, except it is flipped over an axis corresponding to the tape line on the table. After his description using the graph, Noam pantomimed a motion in which he turned the car back toward himself, then backed it back onto the line. In Noam's motion with the car, he turned the car forward away from himself and then backed it towards himself and onto the line. In the figure below, there is a comparison of Noam's initial motion of the car (Figure 5a) with his re-enacted motion of the car (Figure 5b), and the path he traced on the graph with the cursor (Figure 5c).

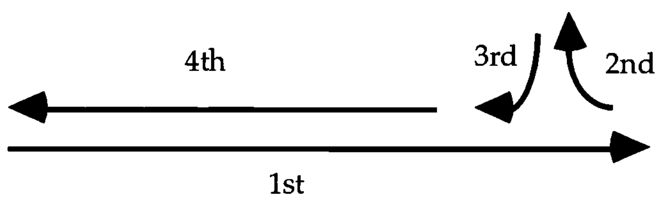


Figure 5a
Noam's initial motion of the car (from Figure 3)

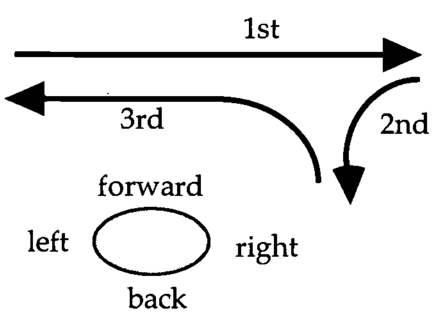


Figure 5b
Noam's re-enacted car motion

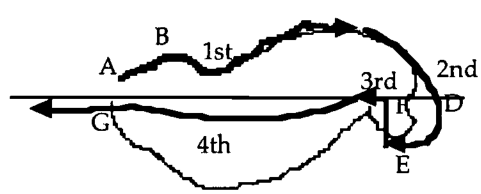


Figure 5c
Noam's motion of the cursor over Graph 1

Noam used Graph 1 not simply to help himself to remember the motion he had made, but as a tool to re-imagine his motion. His motions with the cursor along the graph allowed him to re-animate the graph, which was static, in that it was a fixed display, even though it was used to represent dynamic events. The type of graph Noam was working with, a velocity vs. position graph, has no overt time representation, unlike many graphs used to represent dynamic events. Thus, Noam's cursor motions can be seen as not only re-animating a static graph, but also as adding the time dimension back into this representation of motion.

In this Episode, Noam's gestures were for the most part iconic representations of motion, pointing gestures, and a kind of dynamic pointing gesture accomplished with the mouse and cursor on the computer screen. These were not the usual static pointing gestures, but involved tracing a motion on the computer screen, and thus have much in common with the "tracing points" described by Crowder (1996) in young students' talk about science. Crowder described the way the middle school students in her study used pointing gestures which traced out paths in space to, for instance, trace the path that light rays would follow across the top of an object casting a shadow. Like the tracing points used by the students in Crowder's study, Noam's gestures with the mouse and cursor appear to provide him with new information, as when he simultaneously makes a curved motion with the cursor while completing the sentence "It seems like I tu:rned" <8>, and, in <8.1>, when Noam paused and moved the cursor upward immediately before saying that the car then "backed up" (<8.2>). Noam's motions of the cursor provided a bridge between the static graphical representation and the actual motion he made with the car. Thus, these tracing points, unlike those described by Crowder, as these are used to link a representation with an actual motion.

<u>Episode 2 -- "This here indicates stop because it doesn't *go* like that one"</u>

38:00 - 39:07

<u>Overview.</u> In this Episode, which immediately follows Episode 1, Noam moves the car across the table along the tape line, stopping several times. He tries to associate the stopping motions with particular points on the graph, and this causes him to change the way he interprets graph shapes.

<u>Text.</u>

The interviewer explains that the motion detector can more easily read the location of the car if the car faces the motion detector during it's motion.

TRIAL: Noam moves the car as depicted below:

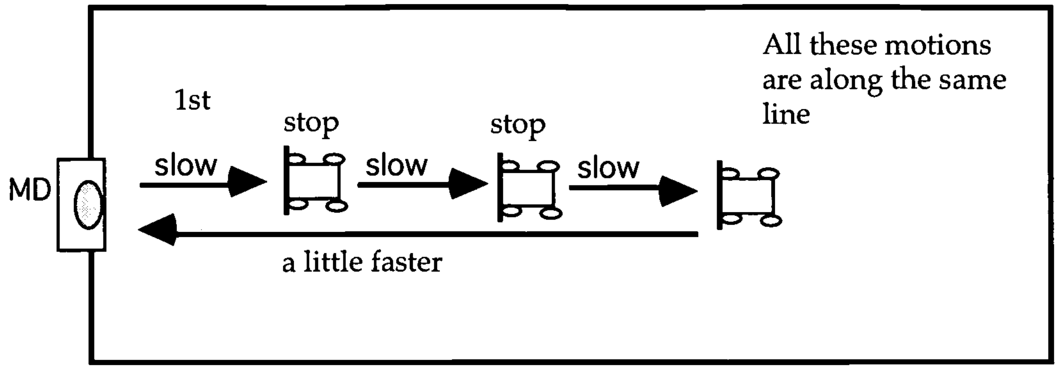
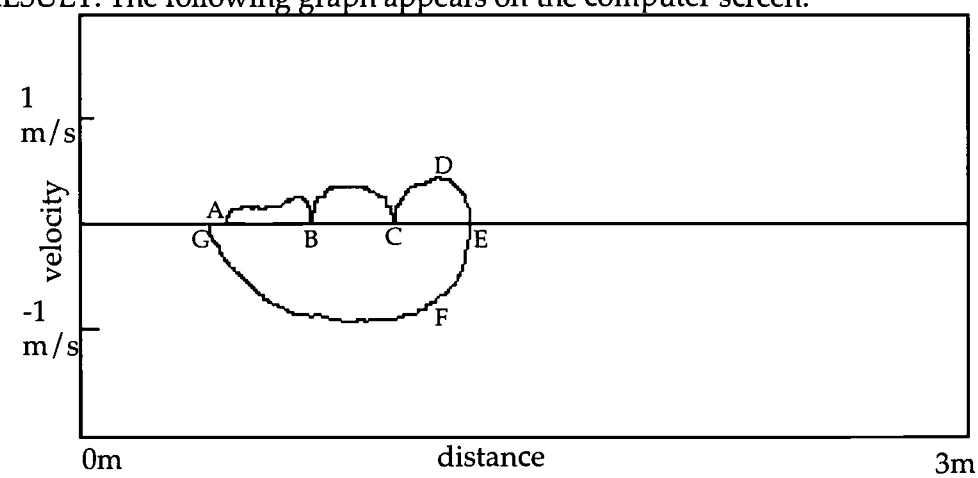


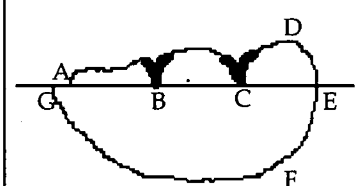
Figure 6

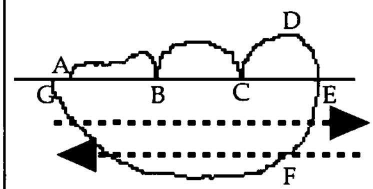
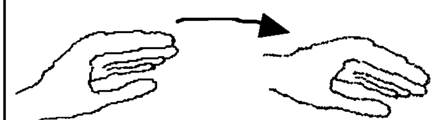
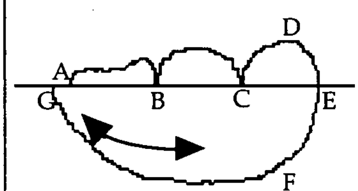
RESULT: The following graph appears on the computer screen:



Graph 2

<11> Noam: How come it [*the graph*]...? [*laughs*] It's off of [*inaud.*]	
<12> Int: Yeah, this one is surprising, too.	*Noam sits down in front of the computer and looks at the screen.*
<13> Noam: OK. I started here.	*Puts cursor at G on Graph 2.*
<13.1> Came here, stopped.	*Moves cursor horizontally to B, stops it.*

<13.2>	Came here, stopped.	*Moves cursor horizontally to C, stops it.*
<13.3>	Came here.	*Moves cursor horizontally to E, stops it.*
<13.4>	Obviously I stopped there,	*Indicates point E with cursor.*
<13.5>	because it didn't,	*Moves cursor along axis from E to G, while tilting his head to his left.*
<13.6>	come back.	*Turns to Interviewer.*
<14> Int:	Okay. So, how did you know - how can you tell from here [*Graph 2*] which places you stopped at?	*Points to computer screen.*
<15> Noam:	[*pause*]	*Moves cursor back and forth along x-axis.*
<15.1>	This here,	*Points to one of these two points on the computer screen:* 
<16> Int:	That there?	
<17> Noam:	indicates stop,	*Hand gesture* 

<17.1>	because it doesn't <u>go like that one</u> [*Graph 2 below the x-axis*].	 *Moves finger across screen along path indicated by dashed line. Turns to Interviewer.*
<17.2>	It's [*Graph 2 at B or C*] not a straight line.	*Gestures with hand as below:* 
<17.3>	Even though that one's not straight but...	*Moves cursor back and forth on screen as indicated below:* 

<u>Interpretive Notes</u>: Noam once again (See <11>) seemed surprised by the graph he made: "How come it [*the graph*]...? (laughs) It's off of," perhaps because the graph was not a straight line, even though his motion was along a straight path. Noam tried to interpret the graph anyway, and began, in <13>, by choosing a starting point for the graph at point G[3] . In <13.1> through <13.3>, Noam moved the cursor along the x-axis, stopping it at points B, C, and E. Instead of trying to follow the graph line with the cursor, Noam stayed on the x-axis with the cursor, just as he had stayed on the tape line on the table with the car. While he took the shape of his motion from his memory of his motion of the car, instead of following the line of the graph, the places he stopped at were determined by the shape of the graph.

In <13> through <13.3>, Noam describing his own motion using the first person: "I started here. Came here, stopped. Came here, stopped. Came here." Yet the "here" he was referring to was in each case a point on the graph indicated by the cursor as he moved it along. Noam was neither just describing his memory the motion of the car, nor just describing the motion of the cursor along

[3]From observing the graph alone, we assume that the point where the graph starts is point A, because that is where the positive velocity line begins. Perhaps Noam was slightly careless in his indication of a starting point.

the graph. His language and gestures fused elements of both the remembered car motion and the current cursor motion (Nemirovsky et al., 1998). In this way, Noam's use of tracing point gestures took on new meaning. He did not simply indicate a graph line, but at once a graph line, a motion along that line, and a motion of the car which created that line. In this way, Noam's use of the cursor resembles gestures made by scientists working with a graph of the state of a diluted antiferromagnet (Ochs et al., 1994). These scientists used their hands to imagine pathways in the graph of the state of this substance, which helped them to envision the kinds of changes of state with temperature or magnetic field which the graph indicated. The authors of this study called these trips through the graph interpretive journeys to highlight the sense in which motion along a graph was used to imagine change over time. Similarly, Noam's movement of the cursor became a means of making interpretive journeys in the graphical space, to explore the meaning of the graph and its relation to various motions.

Noam had made two deliberate stops of the car, and he readily (and correctly) linked these stops with the two dips in the graph at points B and C, which are easily recognizable features on this otherwise smooth graph. He considered the point E a little longer, and argued that he "[o]bviously" stopped at E "because it didn't, [*moves cursor along axis from E to G, while tilting his head to the left*] come back," (<13.4>). The graph's shape at point E was different from its shape at points B and C, where the interruption of a rightward motion is recognizable in the interruption of a line moving left to right. Point E was a stop between moving rightward and moving back to the left, and we will see Noam return to the interpretation of this point in the next episode.

Next, the Interviewer asked Noam how he could tell from the graph where he had stopped with the car (<14>), and Noam pointed out either point B or C on the graph, saying: "This here", gesturing the shape of the graph near that point, "indicates stop" <15> - <17.1>. He continued: "because it [*Graph 2 at B or C*] doesn't *go* like that one [*running his finger right and left across Graph 2, from E to G*]," suggesting that the graph near B or C indicates stop because it is different from the graph from E to G, which "go[es]". Noam's argument seems at first to be specious: the graph at B or C indicates stop, because it doesn't go. However, his gestures tell another story. As Noam says that the graph at B or C indicates stop, he gestures with his hand a curve which resembles the graph as it approaches point B or C, indicating that this curve shape indicates stop to him. When Noam continued by arguing that the graph at B or C doesn't "go" like it does from E to G, he once again used a gesture, this time running his finger right and left across the graph, suggesting which portion of the graph he meant, and at the same time demonstrating how this line "go[es]", or sweeps across the screen without interruption (<17>). Noam's hand gestures involved a stop in one case, and a smooth, uninterrupted motion, in the other. With these gestures, he imitated the shape of the graph, and at the same time explored the kind of motion which made the graph.

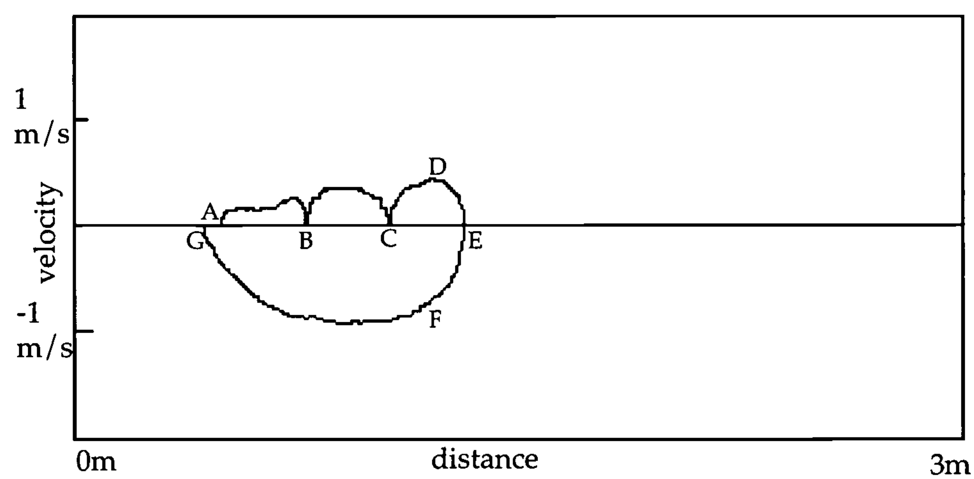
In a velocity vs. position graph, with the position axis as the horizontal, or x-axis, movement along this axis corresponds to movement along the table. In other words, if a graph line travels from left to right along the computer screen, it indicates a motion from left to right along the table (from close to the motion detector to far from the motion detector). When this left to right motion is interrupted, the graph line on the screen is interrupted, and drops down to the axis, indicating zero velocity. Because of this correspondence between the graph and the motion, Noam's movements of the cursor, and his tracing of graph shapes with his hand are useful for finding out about what it is like to move along the graph, and thus can give him information about the motion that created the graph. When a left to right motion is suddenly interrupted by the line going down to the axis, at B and C, the shape itself indicates this interruption and suggests the stopping motion. When the line from about E to about G goes smoothly along without interruption this indicates an uninterrupted motion, that just "go[es]".

Episode 3 -- "No matter how fast it is, that [hand] always stops before coming by it again [moves hand back to the left]"

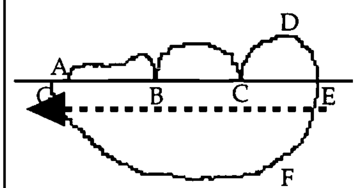
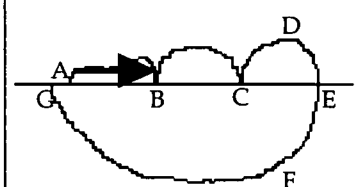
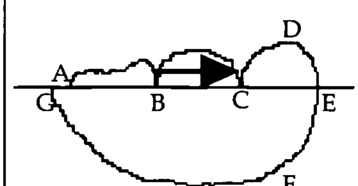
39:08 - 41:50

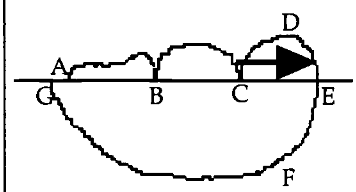
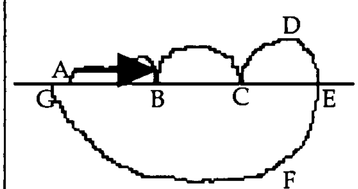
Overview: In this Episode, which immediately follows Episode 2, Noam addresses a contradiction between his belief than any object must stop between a forward and a backward motion and what Graph 2 indicates in the region of point E. The following comments of Noam and the interviewer refer to Graph 2 (See below) from Episode 2, which he created by performing the motion shown in Figure 5 (see previous Episode).

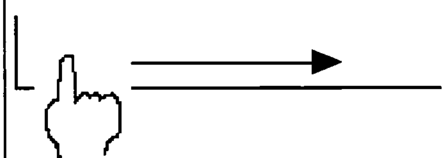
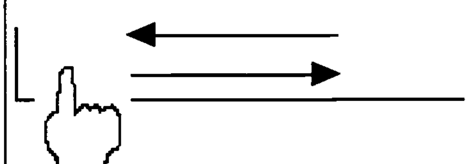
Text:

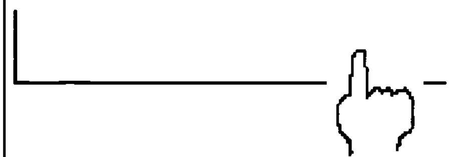
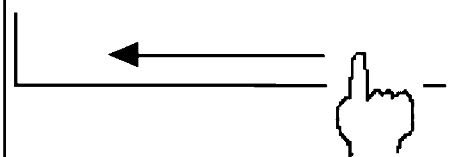
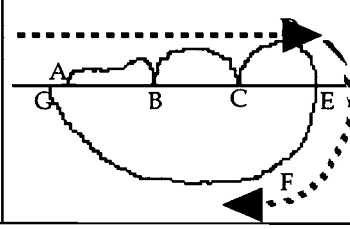


Graph 2

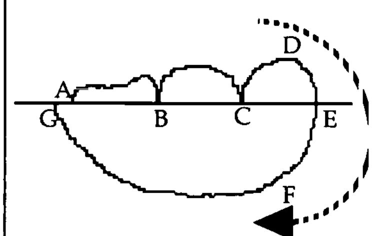
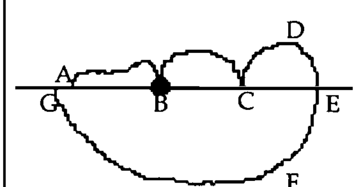
<18> Int:	When you got back here [*to the computer*] you said how come? And then you didn't finish that [*Episode 2, <11>*].	*Noam moves the cursor back and forth horizontally across Graph 2, a little above the x-axis.*
<19> Noam:	Well, I think it was 'cause of the *lines.*	*Runs finger across graph on computer screen:* 
<19.1>	How they weren't straight.	*Gestures with his chin at the car table.*
<20> Int:	How they weren't straight again. Okay.	
<21> Noam:	So [*quietly, to himself*] I came here	*Moves cursor along x-axis to B:* 
<21.1>	stopped here,	*Stops cursor at B.*
<21.2>	stopped here,	*Moves cursor to C and stops it:* 

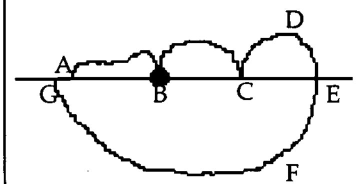
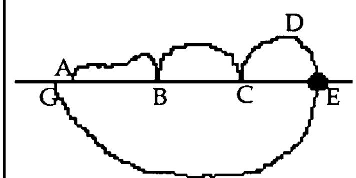
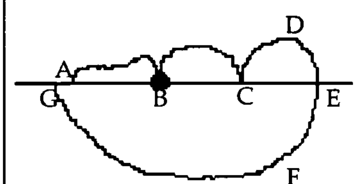
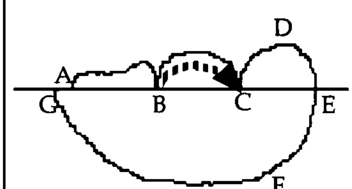
<21.3>	stopped here.	*Moves cursor to E and stops it:* 
<21.4>	But [*to Interviewer*] <u>how come, u:m,</u>	*Looks toward the car table and then returns gaze to computer screen.*
<21.5>	when you sto -	*Returns cursor to A, and then moves it from A to B:* 
<21.6>	when you <u>touch something</u>	*Moves hand from left to right, a few inches above the table:*  

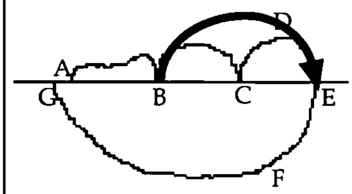
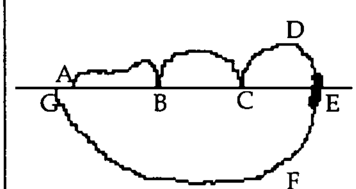
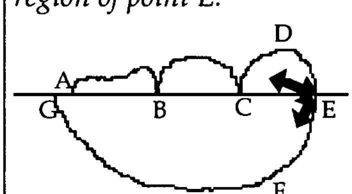
<21.7>	when you touch something, no matter -	*Turns toward interviewer and runs flat hand along table surface from left to right.*
<21.8>	it's like if you throw something	*Makes throwing motion.*
<21.9>	or if you're running something,	*Moves hand left to right as if moving the car on the table.*
<21.10>	and say you go like that	*Moves hand left to right and back to the left along table surface.*
<21.11>	when I go like this,	*Moves R index finger from left to right along table surface:* 
<21.12>	[*pause*]	*Stops R finger and points at it with left hand:* 
<21.13>	no matter how fast it is,	*Fast motion:* 

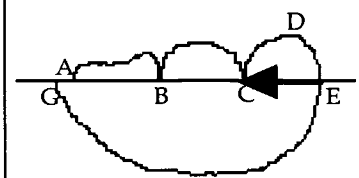
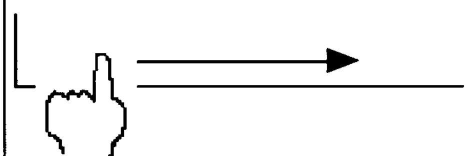
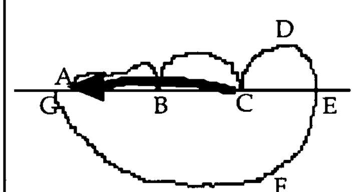
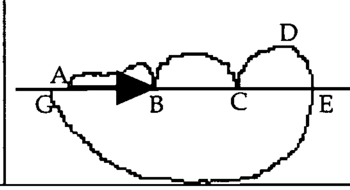
<table>
<tr><td><21.14></td><td>that [hand] always stops</td><td>Taps table at right with finger:
</td></tr>
<tr><td><21.15></td><td>before it comes by again.</td><td>Moves finger back to the left:
</td></tr>
<tr><td><22> Int:</td><td>Oh. OK.</td><td></td></tr>
<tr><td><23> Noam:</td><td>But that doesn't show that-</td><td>Points to graph on computer screen.</td></tr>
<tr><td><23.1></td><td>it just <u>shows he:re,</u></td><td>Traces this path on computer screen with finger

</td></tr>
</table>

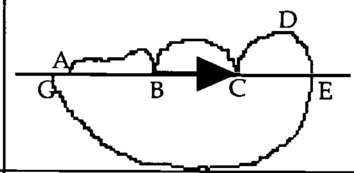
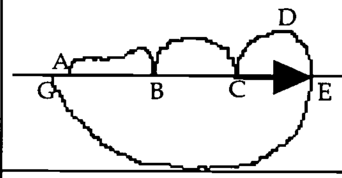
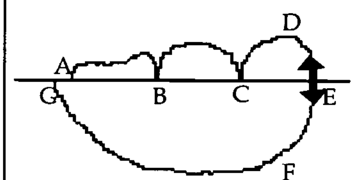
<23.2>	turning around.	*Continues motion in space, turning toward Interviewer at the end:*  
<24> Int:	So you...	
<25> Noam:	Cause I went like this...	*Moves hand from left to right as if holding car in front of himself.* 
<25.1>	And no matter how fast you're moving,	*Moves hand fast left to right to left several times between himself and table, as if holding car in front of himself. Simultaneously shakes head side to side.*
<25.2>	like, I'm moving this arm right now,	*Continues to move hand right to left to right to left.*

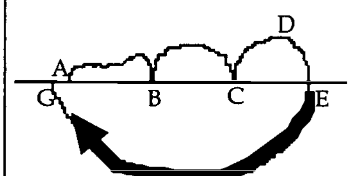
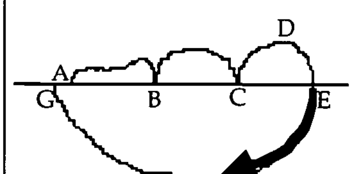
<25.3>	it has to stop one way	*Moves hand to right, stops it, and points at it with the other hand.* 
<25.4>	before it goes to another way.	*Moves right hand back to the left.*
<26> Int:	And it - so - and it doesn't look like it stops there?	*Points to the computer screen.*
<27>Noam:	No. It looks like it's turning.	*Path traced by finger on computer screen.* 
<28> Int:	Okay. Okay. So - but - but - but over - over there it looks like it stops.	*Moves cursor to indicated point on graph:* 
<29> Noam:	Yeah. It looks like it stops,	*Nods head.*
<29.1>	and it continues again.	*Hand moves left to right on table, as if holding car.*

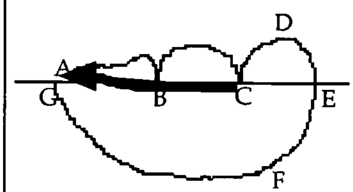
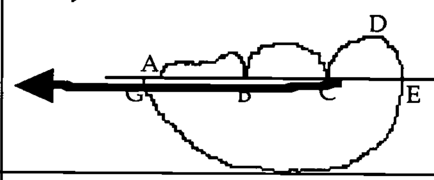
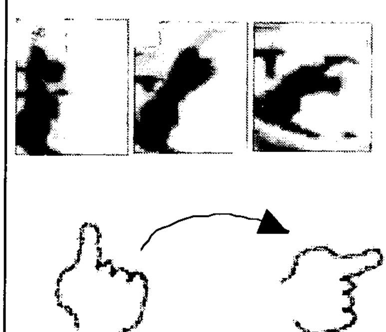
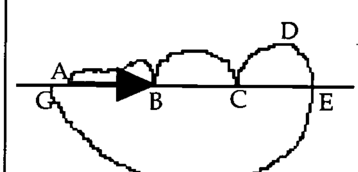
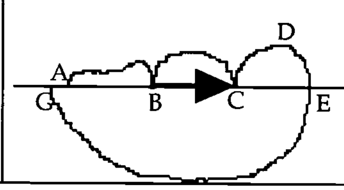
<30> Int:	Okay. And... okay. So, what - can you tell me what it is that's different about this.	*Places cursor at indicated point:* 
<30.1>	as opposed to that?	*Places cursor at indicated point:* 
<30.2>	[*Pause*]	*Returns cursor to point B on the graph.*
<31> Noam:	This one stops	*Points to point B with his finger.* 
<31.1>	and starts, starts all over again.	*Moves finger on graph on computer screen:* 

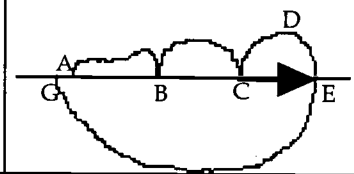
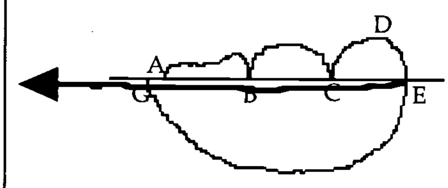
<32> Int:	It stops and starts.	*Moves cursor on computer screen, ending in <29>.* 
<33> Noam:	Stops completely,	*Places hand on table, as if holding car, and keeps it still.*
<33.1>	and starts all over again.	*Moves hand to the right and then stops it.*
<34> Int:	OK, and..	
<35> Noam:	And this one,	*Places hand on table again, back to the left,*
<35.1>	I'm surprised - what surprised me is that	*Points to computer screen, presumably to point E, where the Interviewer had placed the cursor.* 
<35.2>	in this area [*pause*]. Maybe it would	*Places hand on mouse. Keeps cursor on point E, where Interviewer left it.*
<35.3>	[*pause*]	*Moves cursor around tentatively in region of point E.* 
<36> Int:	Useful [*inaud.*] in that area?	*Noam continues tentative cursor motions.*

<table>
<tr><td><37> Noam: [pause] So, maybe</td><td>
Cursor motion on screen</td></tr>
<tr><td><37.1> If I'm going,</td><td>Places left hand flat on table. Continues to hold mouse with right hand and gaze at computer screen.</td></tr>
<tr><td><37.2> the car's going here,</td><td>Moves left index finger along table to the right, and stops it. Gaze at computer screen.
</td></tr>
<tr><td><37.3> stops</td><td>Drops left wrist, leaving left finger on table.</td></tr>
<tr><td><37.4> here</td><td>Moves cursor to point A:
</td></tr>
<tr><td><37.5> comes here, stops</td><td>Moves cursor to B and stops it:
</td></tr>
</table>

<37.6>	then comes here, stops	*Moves cursor to C and stops it:* 
<37.7>	then comes here	*Moves cursor to point D and stops it:* 
<37.8>	Oh, maybe that should stay like that	*Moves cursor in area of E:* 
<37.9>	because, uh, I'm stopping,	*Moves hand above the table (shaped as if holding the car) from left to right a few inches and then stops hand:* 
<37.10>	but then I continue.	*Makes small upward hand motion and then moves hand to the right again. Turns gaze toward Interviewer.*

<37.11>	on the same road.	*Moves hand back toward the left. Returns gaze to the computer screen at the end of this motion:* 
<37.12>	So, maybe that's why it shows	*Returns hand to mouse.*
<37.13>	there [*pause*] 'cause I'm going on the same road.	*Moves cursor along graph towards G:* 
<37.14>	Back-	*Returns cursor to point E.*
<37.15>	just back	*Moves cursor part of the way from E towards G:* 
<38> Int:	Okay. And, and, and - and then why does it [*E*] look different from the other stops [*B and C*]?	*Points to computer screen.*

<39> Noam: [*pause*]	*Moves cursor back towards A:* 
<39.1> 'Cause the other stops don't go back.	*Moves cursor from area of E back to the left:* 
<39.2> And I continue to go to the same pl-, uh	*Hand motion, turning toward Interviewer at the end:* 
<39.3> - stop here	*Moves cursor to B:* 
<39.4> and go to the same direction,	*Moves cursor to C:* 

<39.5>	stop here and go the same direction.	*Moves cursor to E:* 
<39.6>	And over here,	*Small movement of cursor to indicate point E:*
<39.7>	I stopped and went to the opposite direction.	*Moves cursor back to the left:* 

Interpretive Notes: In this episode, Noam brought up the contradiction he perceived between what he knew about motions that change direction and what he saw on Graph 2 at point E. Noam described a rule he believed: that any object must stop at some point between moving one way and moving back the other way (See <25>-<25.4>). However, the graph (Graph 2, area near E) of Noam's forward and back motion of the car didn't seem to him to show this stop between forward and backward motion. According to Noam, "[i]t just shows here [*running hand horizontally along computer screen*], turning around [*runs hand along downward curve on computer screen, curving to the left, then continues motion, horizontally, into space*]" (<23.1-2>).

It is surprising that Noam at this point described point E (on Graph 2) as indicating turning around, which suggests his earlier interpretation of the graph as a 2-dimensional trace of the car's motion. In the previous Episode, Noam had said that he had stopped at point E, arguing that the graph at E indicates a stop in terms of the experience of moving along the line: "it didn't [*moves cursor along axis, from E to G, while tilting head to the left*] come back," (see <13.5-6>). Noam's description of the graph at point E indicating turning around suggests that he recognizes that the graph looks significantly different at point E than it does at points B or C. This difference between a stop which interrupts or ends a motion in one direction, as opposed to a stop which occurs along with a change in direction, has been noted by students of mathematics and physics whom I have worked with. This difference is made especially salient in the phase space graph of these different kinds of stops.

When Noam finally concludes that "maybe that [E] should stay like that, because, uh, I'm stopping , but then I continue on the same road," (see <37.8-10>) to make the graph shape at point E, he points out the critical difference between the points B and E. The motion of the car after the stop at point E is back to the left, whereas the motion after the stop at point B continues to the right. Thus, the same is true of the motion of the graph line: after point E, it comes back to the left, going over some of the "same road" in a sense. Noam's concern about the graph at point E indicating a turn has allowed him to puzzle out this distinction between two kinds of stops: an interruption in a forward motion, and a stop that occurs when an object changes direction. The latter can be considered both a stop and a turn, because there is both a pause in motion and a change in direction; and Noam's relation of the graph at point E to the trajectory of the car has allowed him to see this.

Gestures play an important role in this conflict. Noam repeatedly uses gestures to act out the scenario of moving left to right to left, breaking it up into pieces (see <21.10-15>, <25-25.4>), to show exactly where the stop between forward and backward motion must occur. Noam continues to use gestures in his answer to the Interviewer's question about the difference between Graph 2 at B and at E (see <30 - 30.1>). In Noam's answer, he explicitly connected his gestures on the graph itself with gestures on the table indicating motions of the car. He described what happens at B with hand motions along the graph near B: "This one [*points to B*] stops and starts - starts all over again [*traces from Graph 2 at B upward and to the right*]," (<31-31.1>). Immediately following this, he acts out the motion of the car on the table: "Stops completely [*moves hand left to right, on table and stops*], and starts all over [*continues motion to the right*]," (<33-33.1>). Noam's gestures make explicit his connection between a motion along the graph line from point B upward and to the right and a motion on the table from a stopped position to the right.

Once he has connected the motion of the car with movement along the graph at point B, Noam is able to make a similar connection between the shape of Graph 2 at E and the motion of the car. A few lines after the quote in the previous paragraph, Noam suggests that maybe there is no problem with the shape of the graph at E after all: "Oh, maybe that [*moves cursor over area of E*] should stay like that," (<37.8>). He explains this suggestion in terms of the connection between the motion of the car on the table and the motion along the graph. First, he describes the motion of the car on the table that created the graph near point E: "I'm stopping [*moves hand left to right, stops it*], but then I continue [*small up and down hand motion*] on the same road [*moves hand from right to left*]," (<37.9-11>). Then Noam connects this to the motion along the graph line: "So, maybe that's why it shows there [*moves cursor, from ~E to G*], cause I'm going on the same road. Back - just back [*moves cursor from E towards G again*]," (see <37.12-15>). Note that Noam's cursor motion reflects the motion he made with the car, rather than the turning motion he mentioned earlier in this episode. The connection he has built between motion of the car on the table and motion along the graph line has allowed him to make sense of the dilemma of the meaning of the graph at point E. Noam's sense of motion left-to-right on the graph indicating motion left-to-right on the table and the reverse, which remains from thinking of the graph as a trajectory, has helped him to connect these motions on the graph with motions on the table.

Discussion

We have seen Noam use gestures to join his experience of moving along a graph with the events which created it. Gesture can have a role in both imagining the experience of moving along a graph and in re-imagining the events the graph represents. Gesture is a rarely explored area of students' expression in mathematics education. In this paper I argue that gesture can play a crucial role in imagining or re-enacting the events that created a graph, particularly in the cases where students are investigating graphs of their own motions.

One type of gesture that Noam frequently used in this interview, is the tracing out of curves on the computer screen, done through the use of a computer mouse. Noam often began his story-telling about a graph by moving the cursor along the graph, which indexed his story, as well as allowing him to experience the shape of a particular line. He also frequently used hand gestures, on and off the computer screen, to trace graph shapes, and to help to infer the motion that created them. For Noam, imagining drawing the line of a graph, or acting out that drawing motion in space, helped him to connect visual attributes of a graph shape, such as smoothness, with attributes of the motion that created it, such as continuity.

A second kind of gesture Noam used in the interview is hand movements through space, with or without a toy car in his hand. Noam initially made these motions in front of the motion detector to create graphs of the car's motion. Later, he re-enacted real or hypothetical motions of the toy car without holding the car in his hand. Sometimes Noam used these gestures to slow down a motion, or to play it out step-by-step, to help understand better what occurred and how it related to the resulting graph. These re-enacting gestures helped Noam to think through, as well as express, his arguments the relation of the car's motion to the graph. For instance, the re-enactment of movement from left to right to left in Episode 3 allowed Noam to indicate the point in the motion where the hand changes direction. Since he was able to slow down the motion in this way, this point became an object to look at and consider in a way that it wasn't beforehand. Noam's re-enactments share much in common with Crowder's students' running of models with their gestures (1996). Just as Crowder found that students used gestures to actively run models of the solar system and to draw new conclusions from this thinking-thorough in the moment, Noam also uses his gestures to run models of linear motion and to better understand its limitations.

The context of this interview differs from that described by Crowder (1996), in that the explicit purpose of Noam's activity is the connection of a visual representation of motion with the motion that created it. Thus, Noam's use of gestures is directed towards this goal. Gestures that trace a line shape, or re-enact a motion, can help to connect the visual attributes of the graph with physical motions, becoming part of the story of the graph, and part of the fusion of the shape of the graph with the motion which created it. Noam's line tracing gestures allowed him to experience the shape of a line by moving the cursor along it, by feeling this motion with his hand, and by watching the motion of the cursor on the screen. These gestures allowed Noam to make and support hypotheses about the motion of the car that produced a particular graph shape.

Conclusions

In these episodes, Noam, a student with a limited mathematical background, was able to use his intuitions about how objects move to learn to interpret velocity vs. position graphs, even though these graphs are often considered more difficult to understand than temporal graphs. Through Noam's use of gestures, intuitions about body motion and drawing, and his ability to use the graph to tell the story of his motion, he created a powerful understanding of this unconventional graph.

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